consistent variations due to the geometric parameters are evident in the scatter about the faired curve. Hence, it is concluded that the curve is a good statistical average with the scatter being similar to results obtained when investigating other viscous phenomena.

The curve of Fig. 1 can be used directly to predict the span efficiency of wings, or it can be used as a guide to extrapolate results from wind tunnel tests to full scale. It is especially useful in this regard when model data are available on an actual design where some of the leading-edge suction may be eliminated. For example, fuselages, nacelles, or other components may blanket the wing leading edge locally, thus preventing the possibility of developing suction in such regions.

Above a leading-edge-radius Reynolds number of 100,000, it has become customary to use $R = 0.95$. Upon checking, it was found that this value yields span efficiencies that closely agree with an empirical method used by Sheridan[1] for full-scale aircraft.

The span efficiency e may be computed from Eq. (5) as follows:

$$e = \frac{(C_{L\alpha}/AR)}{R(C_{L\alpha}/AR) + (1 - R)\pi} \tag{6}$$

This expression is plotted in Fig. 2. The significance of the results now becomes apparent. For example, for a fixed value of R, e increases as the ratio of $C_{L\alpha}/AR$ increases. Thus, the wing that produces more $C_{L\alpha}$ per unit aspect ratio will have a higher span efficiency. This fact becomes important in design work and makes the selection of a wing planform and airfoil section take on well-defined significance, since the wing planform geometry is implicit in the value of $C_{L\alpha}$.

The approach of this note with suitable modifications could be extended to higher Mach numbers, including supersonic Mach numbers for wings with subsonic leading edges.

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Approximate Method for Calculating the Compressible Laminar Boundary Layer with Continuously Distributed Suction

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CONTROL of the compressible laminar boundary layer by continuously distributed suction may be employed on airfoils either at small angles of incidence to avoid transition to turbulence and thus to reduce skin friction or at larger angles of incidence to prevent separation and thus to increase the maximum lift. The presumed existence of a laminar boundary layer on the impermeable wall is based on the reduction of density with altitude, so that even at high speeds requiring the consideration of compressibility the critical Reynolds number is not exceeded.

Received by IAS November 9, 1962.
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A rational calculation of compressible boundary layers can be achieved only by an approximation of the Pohlhausen type. Whereas for impermeable walls some integral methods are available,[1] no method of general validity for the case of suction or blowing has been established as yet. In a recent paper[2] a method was presented which has, for the present, been restricted to the special case of an adiabatic wall and a Prandtl number of unity, because for this case a universal relationship between the velocity and temperature profiles exists. In this note a brief account of the method is given.

The method is based on the momentum-integral equation with a pertaining compatibility condition at the wall.[3] The assumption for the velocity profile is of a form

$$u(x,y)/U_e(x) = f[\eta(x,y), K(x)] \tag{1}$$

Here $K(x)$ is a shape factor of the profiles and

$$\eta = \int_0^y \frac{\rho}{\rho_e}\, d\left[\frac{y}{\delta_1(x)}\right] \tag{2}$$

with $\delta_1(x)$ denoting a scale factor proportional to the local boundary layer thickness. The velocity profile used here is composed of an exponential term that reproduces the asymptotic suction profile correctly and a sine term that well approximates the flat-plate profile without suction. The same analytic expression had been used by Schlichting[4] for the incompressible case.

After the introduction of Eq. (1) into the momentum-integral equation, the momentum-loss thickness

$$\vartheta = \int_0^\infty \frac{\rho u}{\rho_e U_e}\left(1 - \frac{u}{U_e}\right) dy \tag{3}$$

in the substituted form

$$Z^* = (\vartheta/l)^2(R/l)^2(U_\infty l/\nu_\infty)$$

is calculated from the equation

$$dZ^*/dx^* = F_1(x^*)\cdot G(k,k_1) + F_2(x^*)\cdot k$$

with

$$k = F_3(x^*)\cdot Z^* \qquad k_1 = F_4(x^*)(Z^*)^{1/2} \tag{6}$$

The quantities F_1 to F_4 are explicit functions of the coordinate $x^* = x/l$ along the chord, which implies a dependence on the external velocity $U_e(x)$, the local Mach number $M_e(x)$, the radius of curvature $R(x)$, and, for F_4, on the suction velocity $v_w(x)$. The shape factor $K(x)$ occurring in Eq. (1) is obtained from a relation

$$K = K(k,k_1) \tag{7}$$

Having thus determined the velocity profile for each position x, the other characteristic parameters such as displacement thickness and skin friction are evaluated easily. The temperature profile is connected with the velocity profile through the general relation just mentioned. For incompressible flow, the method reduces to the one given by Schlichting.[4]

The practical computation is complicated by the fact that the quantity $G(k,k_1)$ occurring in Eq. (5) is not an explicit function of k and k_1. Therefore, Truckenbrodt[5] introduced a linearized form of this function into the incompressible method of Schlichting. Employing such a linearization here also, one obtains, instead of Eq. (5),

$$dY^*/dx^* = A(x^*) - B(x^*)(Y^*)^{1/2} \tag{8}$$

with

$$Y^* = \left(\frac{\vartheta}{l}\right)^2\left(\frac{U_e}{U_\infty}\right)^6\left(\frac{R}{l}\right)^2\left(\frac{T_e}{T_\infty}\right)^{[2(2-\varkappa)/(\varkappa-1)]}\frac{U_\infty l}{\nu_\infty} \tag{9}$$

The quantities A and B are explicit functions of $U_e(x)$, $M_e(x)$,

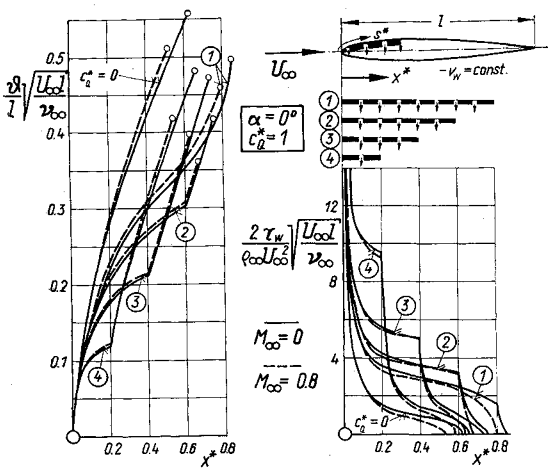

Fig. 1 Laminar boundary layer with continuously distributed suction on profile NACA 0010

$R(x)$, and B also of $v_w(x)$. Having calculated the momentum-loss thickness from Eq. (8), one can evaluate the other boundary layer quantities as just explained.

The method has been checked against special exact solutions, namely, the flat plate and the "similar solutions." In the large range of pressure gradients and suction parameters investigated, good agreement with the exact theory was found.

Furthermore, systematic calculations have been performed for airfoils in the subsonic and supersonic ranges. By means of numerical examples, the influence of the intensity and location of suction on the development of the boundary layer was studied. In Fig. 1 some results are presented for the subsonic airfoil NACA 0010. A given suction quantity

$$c_Q^* = \left(\frac{U_\infty l}{\upsilon_\infty}\right)^{1/2} \int_0^{s_t^*} \frac{-v_w \rho_w}{U_\infty \rho_\infty} ds^* \qquad (10)$$

(where $s^* = s/l$ = coordinate along the contour, and subscript t = trailing edge) is distributed in various ways as indicated in the figure. With decreasing suction zone, the intensity v_w must be increased so that c_Q^* remains constant. For compressible flow, v_w must be increased to compensate for the reduced density ρ_w at the wall. The positions x_{sep}^* of the separation points for the various suction distributions of Fig. 1 are plotted in Fig. 2a. It is demonstrated that by extending the suction zone to the rear part of the profile the point of separation moves in the same direction. For compressible flow, separation occurs somewhat farther upstream. In Figs. 2c and 2d, the same graphs are given for larger angles of incidence. In Fig. 2b, the case is treated in which a suction zone of constant length is situated at various positions along the upper surface. Here, too, suction in the rear part is seen to be more effective in delaying separation than nose suction.

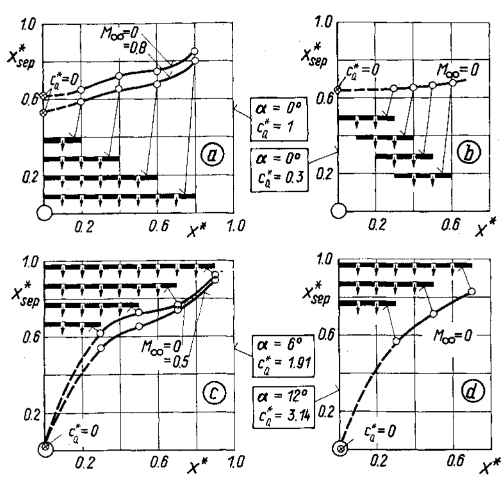

Fig. 2 Shifting of separation point by continuously distributed suction on profile NACA 0010

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Quasi-Steady Aspects of the Adjustment of Separated Flow Regions to Transient External Flows

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The adjustment of separated flow regions with respect to pressure, mass, and heat transfer to transient external flows is investigated using several characteristic times. The adequacy of a quasi-steady treatment is demonstrated, and a quasi-steady solution for the supersonic two-dimensional base pressure problem is presented as an illustration. Experimental data are presented for a transient external flow and compared to the quasi-steady solution for this case. It is concluded that, although initial response to pressure waves is very rapid, the adjustment due to mass and heat transfer is much slower, and, as a result, short-duration experiments on separated flows should come under special scrutiny for correct interpretations of results.

The dependence of wake problems on both the dynamics of the external, nearly isentropic stream and the dissipative mechanism of jet-mixing regions is well established.[1–3]

Received by ARS December 10, 1962. This paper is based on a Ph.D. thesis by Mr. Ihrig entitled "A Study of the Base Pressure Problem for Supersonic Two-Dimensional Flow in Which the Approaching Stream Has Transient Properties," Department of Mechanical and Industrial Engineering, University of Illinois, Urbana, Ill., June 1961.

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